

X847/76/11

Mathematics Paper 1 (Non-calculator)

MONDAY, 12 MAY 9:00 AM – 10:15 AM



#### Total marks — 55

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use blue or black ink.

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#### **FORMULAE LIST**

#### Circle

The equation  $x^2 + y^2 + 2gx + 2fy + c = 0$  represents a circle centre (-g, -f) and radius  $\sqrt{g^2 + f^2 - c}$ . The equation  $(x-a)^2 + (y-b)^2 = r^2$  represents a circle centre (a,b) and radius r.

Scalar product

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$$
, where  $\theta$  is the angle between  $\mathbf{a}$  and  $\mathbf{b}$ 

or 
$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$
 where  $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$  and  $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ .

Trigonometric formulae

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives

f(x)	f'(x)
sin ax	$a\cos ax$
cos ax	$-a\sin ax$

Table of standard integrals

f(x)	$\int f(x)dx$
sin ax	$-\frac{1}{a}\cos ax + c$
cos ax	$\frac{1}{a}\sin ax + c$

# Total marks — 55 marks Attempt ALL questions

- 1. A curve has equation  $y = x^3 2x^2 + 5$ .
  - Find the equation of the tangent to this curve at the point where x = 2.

4

4

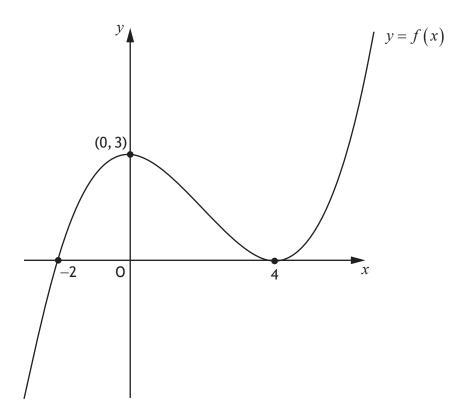
- 2. Find the equation of the perpendicular bisector of the line joining A(1, 4) and B(9, 10).
- 3. Find  $\int \left(\frac{12}{x^2} + x^{\frac{1}{2}}\right) dx$ , x > 0.

4

3

4. Evaluate  $3\log_3 2 + \log_3 \frac{1}{24}$ .

5. The diagram shows the graph of y = f(x), with stationary points at (0, 3) and (4, 0).

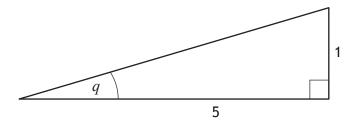


On the diagram in your answer booklet, sketch the graph of y = f(-x) + 3.

2

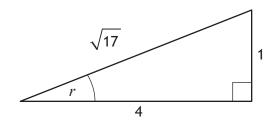
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**6.** The diagram shows a right-angled triangle with angle q.



- (a) Determine the value of:
  - (i)  $\sin 2q$
  - (ii)  $\cos 2q$ .

A second right-angled triangle has angle r as shown.



- (b) Find the value of  $\sin(2q-r)$ .
- 7. (a) Show that (x+3) is a factor of  $5x^3 + 16x^2 x 12$ .
  - (b) Hence, or otherwise, solve  $5x^3 + 16x^2 x 12 = 0$ .
- 8. Given that  $\log_a 75 = 2 + \log_a 3$ , a > 0, find the value of a.

**9.** Find the coordinates of the points of intersection of the line with equation y = x + 1 and the circle with equation  $x^2 + y^2 - 2x + 6y - 15 = 0$ .

4

- The vectors **u** and **v** are such that:

  - the angle between **u** and **v** is 45°.

Find the value of k, where k > 0.

5

11. The equation  $9x^2 + 3kx + k = 0$  has two real and distinct roots.

Determine the range of values for k.

Justify your answer.

4

- 12. Given that:
  - $\frac{dy}{dx} = 6\cos x + 8\sin 2x$ , and  $y = 4 \text{ when } x = \frac{\pi}{6}$ ,

express y in terms of x.

**13.** A function, f, is defined on the set of real numbers.

The derivative of f is f'(x) = (x+5)(2-x).

(a) Find the *x*-coordinates of the stationary points on the curve with equation y = f(x) and determine their nature.

3

It is known that:

- $\bullet \quad f \text{ is a cubic function} \\$
- f(0) < 0
- the equation f(x) = 0 has exactly one solution. The solution lies between -10 and 10.
- (b) Draw a sketch of a possible graph of y = f(x) on the diagram in your answer booklet.

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X847/76/12

Mathematics Paper 2

MONDAY, 12 MAY 10:45 AM – 12:15 PM

#### Total marks — 65

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You may use a calculator.

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#### **FORMULAE LIST**

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Table of standard derivatives

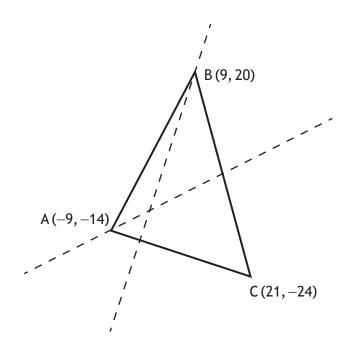
f(x)	f'(x)
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Table of standard integrals

f(x)	$\int f(x)dx$
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## Total marks — 65 Attempt ALL questions

1. Triangle ABC has vertices A (-9, -14), B (9, 20) and C (21, -24).



(a) Find the equation of the altitude through B.

3

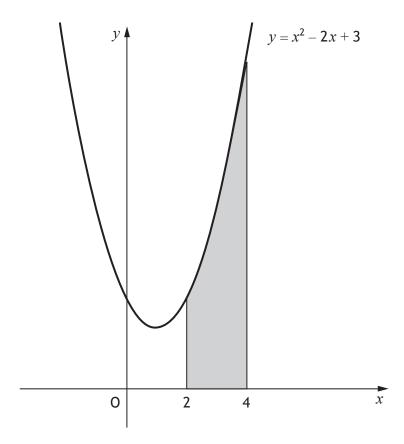
(b) Find the equation of the median through A.

- 3
- (c) Determine the point of intersection of the altitude through B and the median through A.
- 2

**2.** Express 
$$2x^2 + 16x + 5$$
 in the form  $p(x+q)^2 + r$ .

3

3. The diagram shows the graph of  $y = x^2 - 2x + 3$ .



Calculate the shaded area.

4

**4.** A function, g, is defined by  $g(x) = (x-4)^3$ , where  $x \in \mathbb{R}$ . Find the inverse function,  $g^{-1}(x)$ .

3

5. (a) Show that the points A(-3, 2, -1), B(6, -1, 5) and C(12, -3, 9) are collinear.

3

(b) State the ratio in which B divides AC.

1

**6.** (a) Express  $5\cos x - 9\sin x$  in the form  $k\cos(x+a)$  where k>0 and  $0< a<2\pi$ .

1

(b) Hence solve  $5\cos x - 9\sin x = 7$  for  $0 \le x < 2\pi$ .

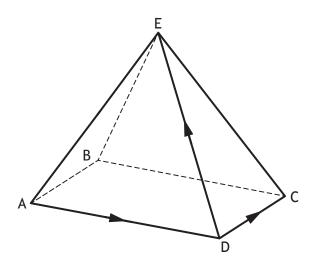
3

**8.** E,ABCD is a rectangular-based pyramid as shown.

$$\overrightarrow{AD} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

$$\overrightarrow{DC} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$$

$$\overrightarrow{\text{DE}} = -4i - 3j + 4k$$



Express  $\overrightarrow{BE}$  in terms of i, j and k.

2

- **9.** A sequence satisfies the recurrence relation  $u_{n+1} = mu_n + 4$ , where m is a constant.
  - (a) The sequence approaches a limit of 10 as  $n \to \infty$ . Determine the value of m.

2

(b) Given that  $u_1 = 19$ , calculate the value of  $u_0$ .

1

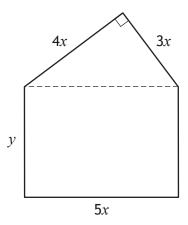
**10.** A hotel owner is designing signs showing the room numbers.



Each sign is a rectangle with a right-angled triangle above it.

The length and breadth of the rectangle are 5x centimetres and y centimetres respectively.

The shorter sides of the triangle are 3x centimetres and 4x centimetres.



The area of the sign is 150 square centimetres.

(a) Show that the perimeter, P cm, of the sign is given by

$$P = 9.6x + \frac{60}{x}.$$

Each sign will be lit using a lighting strip placed around its perimeter.

The hotel owner requires the perimeter, P, of the sign to be as small as possible.

(b) Find the minimum value of *P*.

11. Solve  $3\sin 2x^{\circ} + 4\cos x^{\circ} = 0$  for  $0 \le x < 360$ .

4

- **12.** Functions f and g are defined on the set of real numbers by:
  - $f(x) = x^5 + 3$
  - $g(x) = 1 x^3$ .
  - (a) Find an expression for h(x), where h(x) = f(g(x)).

2

(b) Find h'(x).

2

13. A radioactive substance, which has been collected, decays over time.

The mass of the radioactive substance remaining is modelled by

$$M = 150e^{-0.0054t}$$

where  ${\cal M}$  is the mass, in micrograms, t years after the radioactive substance was collected.

(a) Determine the initial mass of the radioactive substance.

1

(b) Calculate the time taken for the mass of the radioactive substance to decay to 120 micrograms.

4

- **14.** Circle C<sub>1</sub> has equation  $(x+5)^2 + (y-6)^2 = 9$ .
  - (a) State the centre and radius of  $C_1$ .

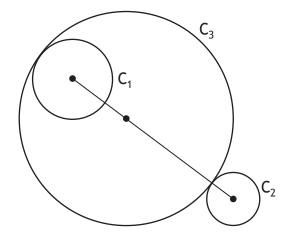
2

Circle C<sub>2</sub> has equation  $x^2 + y^2 - 14x + 6y + 54 = 0$ .

(b) State the centre and radius of  $C_2$ .

2

Circles  $C_1$ ,  $C_2$  and  $C_3$  are touching as shown in the diagram. The centre of circle  $C_3$  lies on the line joining the centres of  $C_1$  and  $C_2$ .



(c) Determine the equation of  $C_3$ .

3

### [END OF QUESTION PAPER]